Chapter 8

Finite Automata with Output

8.1 Moore Machines

Definition: A *Moore machine* is a collection of five things:

1. A finite set of states $q_0, q_1, q_2, \ldots, q_n$, where q_0 is designated as the start state.
2. A finite alphabet of letters for forming the input string

$$\Sigma = \{a, b, c, \ldots\}$$

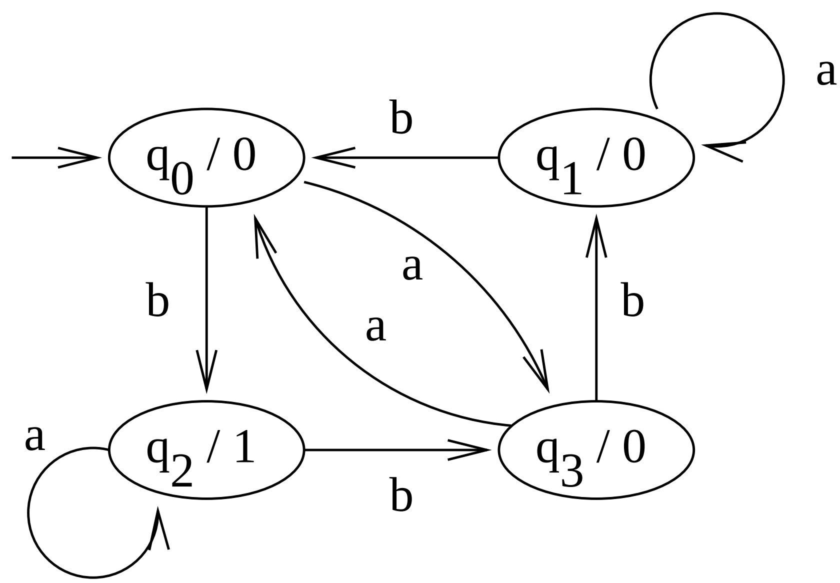
3. A finite alphabet of possible output characters

$$\Gamma = \{x, y, z, \ldots\}$$

4. A transition table that shows for each state and each input letter what state is reached next.
5. An output table that shows what character from Γ is printed by each state that is entered.

Example: Input alphabet: $\Sigma = \{a, b\}$
Output alphabet: $\Gamma = \{0, 1\}$
States: q_0, q_1, q_2, q_3

	a	b	Output
q_0	q_3	q_2	0
q_1	q_1	q_0	0
q_2	q_2	q_3	1
q_3	q_0	q_1	0



On input string *bababbb*, the output is 01100100.

8.2 Mealy Machines

Definition: A *Mealy machine* is a collection of four things:

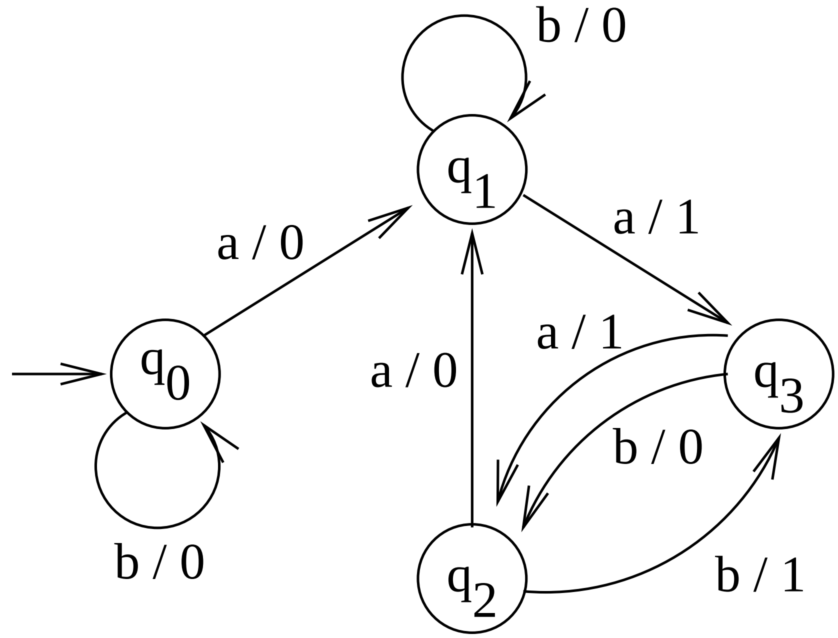
1. A finite set of states $q_0, q_1, q_2, \ldots, q_n$, where q_0 is designated as the start state.
2. A finite alphabet of letters $\Sigma = \{a, b, \ldots\}$.
3. A finite alphabet of output characters $\Gamma = \{x, y, z, \ldots\}$.

4. A pictorial representation with states reresented by small circles and directed edges indicating transitions between states. Each edge is labeled with a compound symbol of the form i/o, where i is an input letter and o is an output character. Every state must have exactly one outgoing edge for each possible input letter. The way we travel is determined by the input letter i. While traveling on the edge, we must print the output character o.

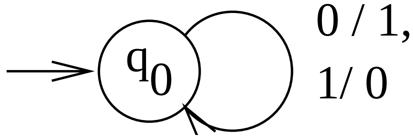
The key difference between Moore and Mealy machines:

- Moore machines print character when in state.
- Mealy machines print character when traversing an arc.

Example: Mealy machine



Example: Mealy machine prints out the 1's complement of an input bit string.
$\Sigma = \Gamma = \{0, 1\}$.



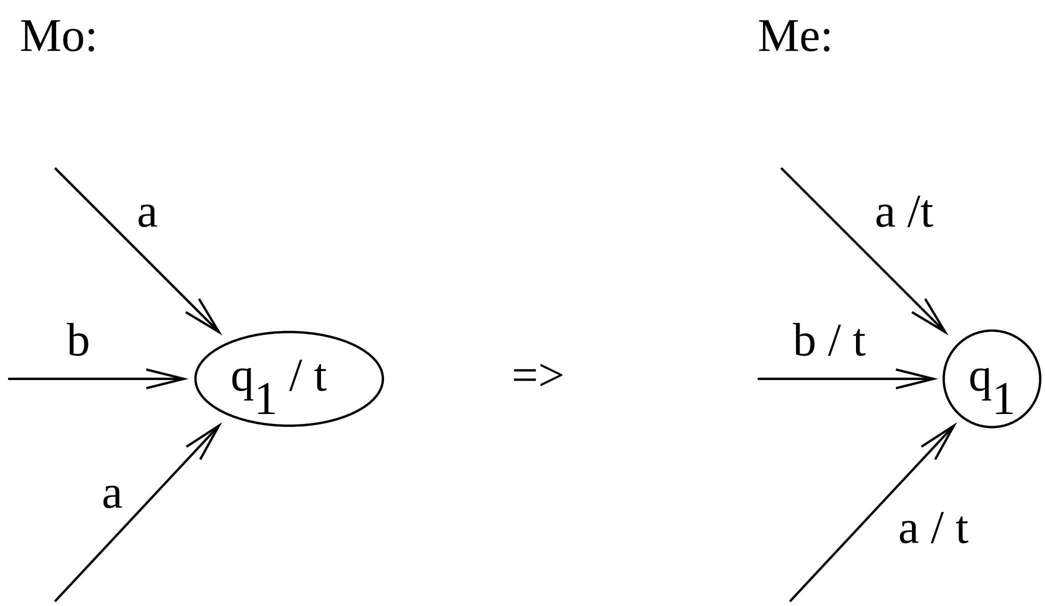
8.3 Properties of Moore and Mealy Machines

Definition: Given a Mealy machine *Me* and a Moore machine *Mo*, which automatically prints the character x in the start state, we say these two machines are *equivalent* if for every input string the output string from *Mo* is exactly x concatenated with the output from *Me*.

Theorem 8 *If Mo is a Moore machine, then there is a Mealy machine Me that is equivalent to it.*

Proof.

- Consider any state q_i of *Mo*.
- Suppose *Mo* prints the charater t upon entering q_i.
- Hence, the label in state q_i is q_i/t.
- Suppose that there are n arcs entering q_i, with labels $a_1, a_2, \ldots, a_n$.
- We create the machine *Me* by changing the labels on the incoming arcs to q_i to a_m/t, $m = 1, 2, \ldots, n$.
- Change the label of state q_i to be just q_i.



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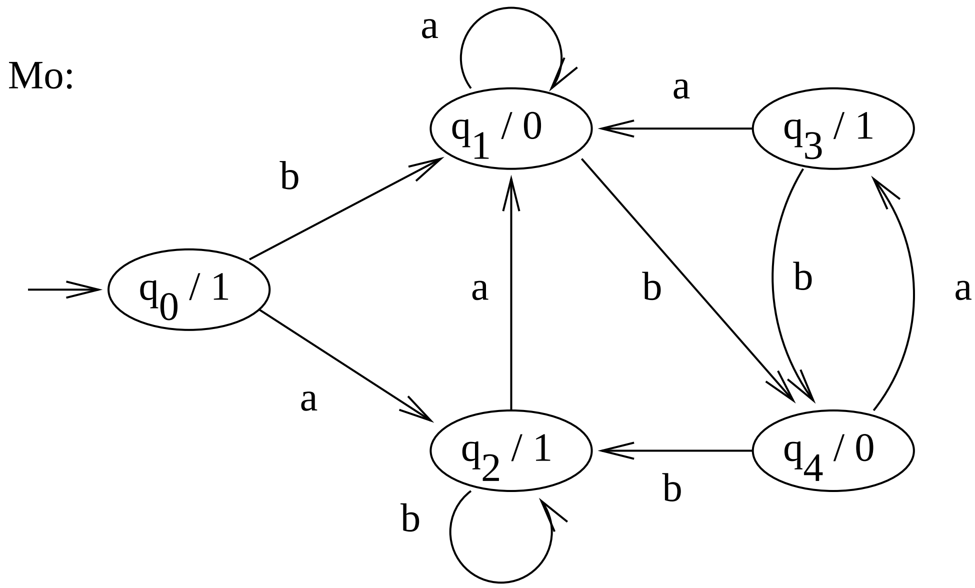
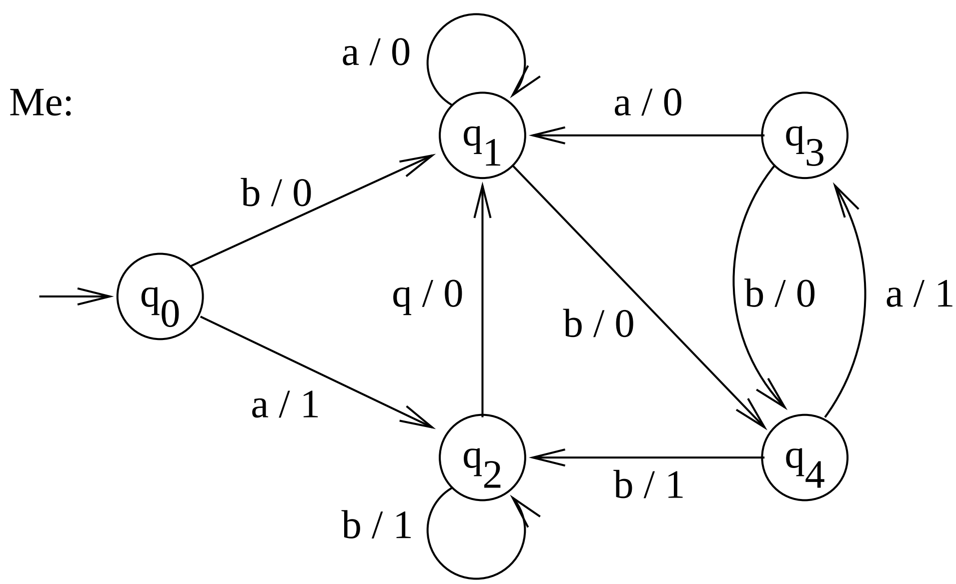
Example: Convert Moore machine into equivalent Mealy machine.

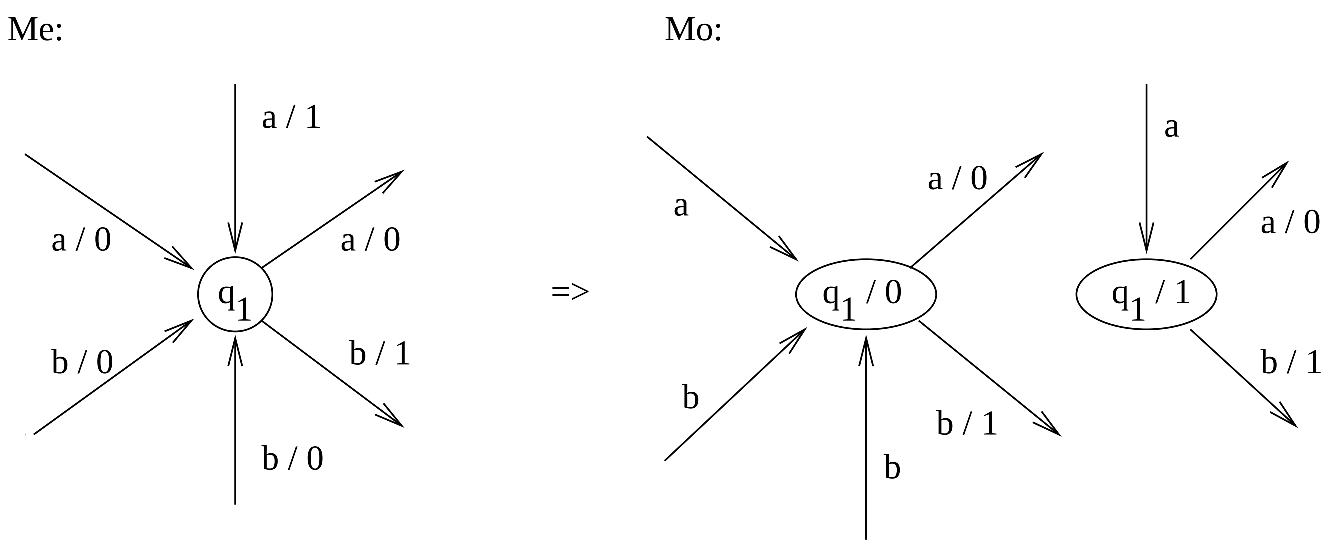




Theorem 9 *For every Mealy machine Me, there is an equivalent Moore machine Mo.*

Proof.

- Consider any state q_i of *Me*.
- Suppose that there are n arcs entering q_i, with labels $a_1/t_1, a_2/t_2, \ldots, a_n/t_n$.
- So if we enter state q_i using the kth arc, we just read in a_k and printed t_k.
- Suppose that among $\{t_1, t_2, \ldots, t_n\}$, there are k different characters; call them $c_1, c_2, \ldots, c_k$.
- To create the Moore machine *Mo*, split the state q_i into k different states; call them $q_i^1, q_i^2, \ldots, q_i^k$.
- State q_i^l will correspond to the output character c_l.
- For each arc going into q_i in *Me* which was labeled with the output character c_l, have that arc in *Mo* go to the state q_i^l/c_l. Label that arc with its input letter.
- For any state in *Me* which has no incoming edges, we arbitrarily assign it any output character in *Mo*.



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Example: Convert Mealy machine into equivalent Moore machine.